Inclusion of a Direct and Inverse Energy-Consistent Hysteresis Model in Dual Magnetostatic Finite Element Formulations

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This paper deals with the implementation of an energy-consistent ferromagnetic hysteresis model in 2D finite element computations. Being naturally vectorial, the hysteresis model relies on a strong thermodynamic foundation and ensures the closure of minor hysteresis loops. The model accuracy can be increased by controlling the number of intrinsic cell components while parameters can be easily fitted on common material measurements. Here, the native $\boldsymbol{h}$-based material model is inverted using the Newton-Raphson method for its inclusion in the magnetic vector potential formulation. Simulations are performed on a 2D T-shaped magnetic circuit exhibiting rotational flux. By way of validation, comparison is made with results obtained by the dual magnetic scalar potential formulation. A very good agreement for global quantities is observed.

Index Terms—**Finite element analysis, Magnetic hysteresis, Newton Method.**

I. Introduction

In the domain of numerical electromagnetism, increasing attention is paid to the modeling of ferromagnetic hysteretic materials with the aim of predicting the iron losses with high accuracy. However, the inclusion of a hysteresis model in a finite element (FE) computation remains challenging due to strong nonlinearities and potential inconsistencies between the input vector variable of the model (magnetic field $\boldsymbol{h}$ or induction field $\boldsymbol{b}$) and the basic variable of the FE formulation.

In this paper, an energy-consistent hysteresis model [1], [2], [4] is incorporated in a 2D FE model with the classical one-component magnetic vector potential (MVP) formulation. First, the hysteresis model is briefly presented in its original direct form, driven by the magnetic field $\boldsymbol{h}$ as input. Since the MVP-formulation has the induction field $\boldsymbol{b}$ as unknown field, the hysteresis model needs to be driven by the variable $\boldsymbol{b}$ instead. This inversion is done with the Newton-Raphson (NR) technique. Finally, FE simulations with the MVP-formulation (inverse hysteresis model included) are carried out on a T-joint of a three-phase transformer and compared with results obtained with the dual magnetic scalar potential (MSP) formulation (direct model included).

II. Energy-Consistent Hysteresis Model

The magnetic hysteresis finds its physical origin at the level of Weiss domains with the pinning effect of Bloch walls around defects in the material structure. The energy-consistent hysteresis model [1], [2] which has similarities with the one presented in [3], is based on the analogy between this pinning effect and the dry friction in mechanics so that it has a simple mechanical representation formed by the parallel connection of a spring and a friction slider. The statistical distribution of the pinning field that is specific to each material and characterizes most of its hysteretic behavior is discretized and incorporated into the model by considering several spring-slider cells connected in series (see Fig. 1). The applied force is analogous to the magnetic field $\boldsymbol{h}$ while the elongation is the magnetic polarization $\boldsymbol{J}$ that is split up in N components spread over each cell ($\boldsymbol{J} = \sum \boldsymbol{J}^k$). Friction sliders, which model the pinning effect, are unlocked when the applied field exceeds a threshold specific to each cell and denoted χ^k. On the other hand, the energy u^k in the springs corresponds to the magnetic energy stored in the material. The springs thus take the reversible part of the material response while the friction sliders take the irreversible one.

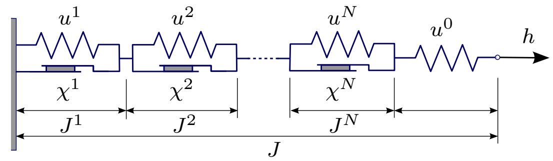

Fig. 1. Mechanical analogy of the hysteresis model with N cells.

The direct $\boldsymbol{h}$-based implementation of the energy-consistent hysteresis model consists in building for each cell a functional Ω^k whose minimization at each instant amounts to finding the material polarization components $\boldsymbol{J}^k$ [2]. This functional reads

$$\Omega^k(\boldsymbol{h}, \boldsymbol{J}^k, \boldsymbol{J}_p^k) = u^k(|\boldsymbol{J}^k|) - \boldsymbol{h} \cdot \boldsymbol{J}^k + \chi^k \, |\boldsymbol{J}^k - \boldsymbol{J}_p^k| \,, \tag{1}$$

where the energy density u^k is defined by the integration of a scalar saturation curve $u^k(|\boldsymbol{J}^k|) := \int_0^{|\boldsymbol{J}^k|} \alpha \, \mathrm{atanh}\,(J'/J_s^k)\, \mathrm{d}J'$, with J_s^k, the saturation magnetic polarization, and α, a parameter inversely proportional to the slope of the curve at the origin. $\boldsymbol{J}_p^k$ represents the magnetic polarization field at previous time step and contains the magnetic history of the material response. The updated values $\boldsymbol{J}^k$ follow from minimizing separately each independent Ω^k (1):

$$\boldsymbol{J}^k = \mathcal{J}^k(\boldsymbol{h}, \boldsymbol{J}_p^k) = \underset{\boldsymbol{J}^k}{\mathrm{argmin}}\, \Omega^k(\boldsymbol{h}, \boldsymbol{J}^k, \boldsymbol{J}_p^k) \,. \tag{2}$$

The magnetic induction field $\boldsymbol{b}$ can then be computed:

$$\boldsymbol{b} = \mathcal{B}(\boldsymbol{h}, \boldsymbol{J}_p^k) = \mu_0 \boldsymbol{h} + \sum \mathcal{J}^k(\boldsymbol{h}, \boldsymbol{J}_p^k) \,, \tag{3}$$

where μ_0 is the magnetic permeability of vacuum.

III. Model Inversion with NR

Taking $\boldsymbol{b}^*$ as a known input value, the Newton-Raphson technique consists in finding increasingly better approximations of the field $\boldsymbol{h}^*$ that verifies $\boldsymbol{r}(\boldsymbol{h}^*) = \mathcal{B}(\boldsymbol{h}^*, \boldsymbol{J}_p^k) - \boldsymbol{b}^* = \boldsymbol{0}$, where $\mathcal{B}$ comes from the direct model (3). Starting from an initial estimate $\boldsymbol{h}_0$, the NR process produces subsequent increment $\Delta\boldsymbol{h}_{i+1}$, and so next estimated magnetic field value $\boldsymbol{h}_{i+1} = \boldsymbol{h}_i + \Delta\boldsymbol{h}_{i+1}$, from linearisation of $\boldsymbol{r}(\boldsymbol{h}_{i+1})$ around the current known approximation $\boldsymbol{h}_i$:

$$\Delta\boldsymbol{h}_{i+1} = \left[\frac{\partial\mathcal{B}}{\partial\boldsymbol{h}}(\boldsymbol{h}_i, \boldsymbol{J}_p^k)\right]^{-1} \cdot \left(\boldsymbol{b}^* - \mathcal{B}(\boldsymbol{h}_i, \boldsymbol{J}_p^k)\right) , \qquad (4)$$

where $\frac{\partial\mathcal{B}}{\partial\boldsymbol{h}}$ is the differential permeability tensor that can be calculated analytically except at angoulous points ($|\boldsymbol{J}^k| = 0$ or $|\boldsymbol{J}^k - \boldsymbol{J}_p^k| = 0$) where it is approximated numerically. The process is repeated until sufficient convergence ($|\boldsymbol{r}(\boldsymbol{h})| < \epsilon$). More details about the inversion will be given in the full paper. In the end, the inverse model can be summarized as follows:

$$\boldsymbol{h} = \mathcal{H}(\boldsymbol{b}, \boldsymbol{J}_p^k) = \arg\min_{\boldsymbol{h}} \left|\mathcal{B}(\boldsymbol{h}, \boldsymbol{J}_p^k) - \boldsymbol{b}\right| . \qquad (5)$$

IV. 2D FE Implementation (MVP-Formulation)

The inverse model (5) is added in the 2D FE method with the classical magnetostatic $\boldsymbol{a}$-formulation, weak form of the Ampere law ($\operatorname{curl}\boldsymbol{h} = \boldsymbol{j}_s$): find the MVP $\boldsymbol{a} = a_z\boldsymbol{e}_z$ such that

$$\left(\mathcal{H}\left(\operatorname{curl}\boldsymbol{a}, \boldsymbol{J}_p^k\right), \operatorname{curl}\boldsymbol{a}'\right)_\Omega = \left(\boldsymbol{j}_s, \boldsymbol{a}'\right)_{\Omega_s} \qquad (6)$$

holds for suitable test functions $\boldsymbol{a}'$, where $(\cdot,\cdot)_\Omega$ denotes a volume integral in Ω of the scalar product of the two arguments, $\boldsymbol{j}_s$ is the imposed current density in domain $\Omega_s \in \Omega$ and homogeneous boundary conditions were considered for the MVP on $\partial\Omega$. In practice, $\boldsymbol{a}$ is discretized with appropriate basis functions that are also used as test functions $\boldsymbol{a}'$ (Galerkin method); the induction $\boldsymbol{b} = \operatorname{curl}\boldsymbol{a}$ satisfies exactly $\operatorname{div}\boldsymbol{b} = 0$.

Because of the hysteretic material behavior, the system (6) has to be solved by time stepping. Starting from a known solution $\boldsymbol{a}_p$ and known material state $(\boldsymbol{b}_p, \boldsymbol{h}_p, \boldsymbol{J}_p^k)$ at the present instant, the solution at the next instant can be obtained by means of an iterative NR scheme:

$$\left(\frac{\partial\mathcal{H}}{\partial\boldsymbol{b}}\left(\boldsymbol{h}_i, \boldsymbol{J}_p^k\right) \cdot \operatorname{curl}\Delta\boldsymbol{a}_{i+1}, \operatorname{curl}\boldsymbol{a}'\right)_\Omega = \left(\boldsymbol{j}_s, \boldsymbol{a}'\right)_{\Omega_s} - \left(\boldsymbol{h}_i, \operatorname{curl}\boldsymbol{a}'\right)_\Omega , \qquad (7)$$

where the differential reluctivity tensor $\frac{\partial\mathcal{H}}{\partial\boldsymbol{b}} = \left[\frac{\partial\mathcal{B}}{\partial\boldsymbol{h}}\right]^{-1}$ emerges. The notation $\boldsymbol{h}_i = \mathcal{H}\left(\operatorname{curl}\boldsymbol{a}_i, \boldsymbol{J}_p^k\right)$ is used for the last computed magnetic field value. Magnetic polarization components $\boldsymbol{J}^k$ are updated thanks to (2) to completely define the new material state and for serving for the next time step.

V. Simulations

A 3-cell hysteresis model fitted on the basis of Epstein measurements carried on the electrical steel M250-50A [2] is considered. The MVP-formulation with the inverse hysteresis model (7) is applied to a simple 2D FE problem: a T-joint of a three-phase transformer (Fig. 2 (left)) [8]. The boundary of the T-joint is considered as a sequence of flux walls Γ_{wi} (magnetically impermeable interfaces) and flux gates Γ_{gi} (perfectly permeable interfaces). FE simulations are also performed using the direct hysteresis model (3) in the dual MSP-formulation, for which $\boldsymbol{h} = -\operatorname{grad}\phi$ is the unknown field (as done in [8] with the Jiles-Atherton hysteresis model). A rotational field is effected by imposing (strongly in the MVP-formulation and weakly in the MSP-formulation) gate fluxes with appropriate phase shift. The magnetomotive forces $\mathcal{F}_1$ and $\mathcal{F}_2$ associated to flux walls Γ_{w1} and Γ_{w2} respectively are shown in Fig. 2 (right). A very good agreement is observed between the results of the two formulations. The b-loci and h-loci for the six points from Fig. 2 (left) are given in Fig. 3.

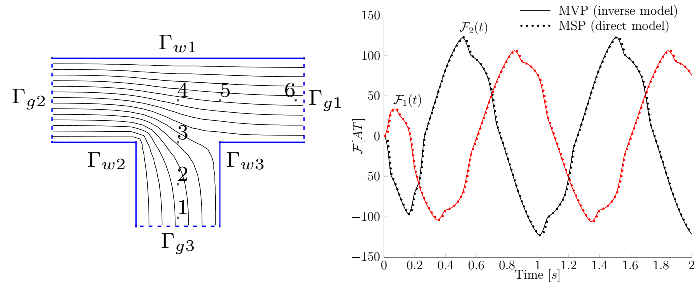

Fig. 2. (left): T-joint transformer with flux lines and location of six points. (right): Magnetomotive forces $\mathcal{F}_1(t)$, $\mathcal{F}_2(t)$ obtained on resp. Γ_{w1}, Γ_{w2}

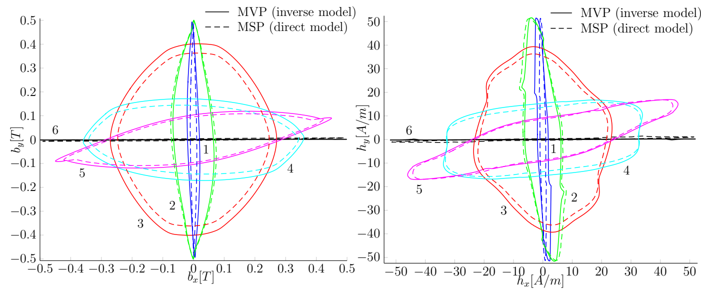

Fig. 3. b-loci (left) and h-loci (right) in the six points defined in Fig. 2 (left).

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